**Numerical Verification**

Verification and validation (V&V) are the most important steps to assess the accuracy and reliability of any numerical simulation.

Figure : Stages of CFD Modeling and the

Conceptual Model (Continuum)

Computational Model (Discertized)

Natural Phenomenon

(Reality)

Validation

Qualification

Verification

Programming

Simplification

Simulation

In simple words in verification correctness of the solution technique used is evaluated and validation is evaluating whether the model is proper representative of the processes involved in the problem. Lax Equivalence Theorem which indicates that for a solver to be stable and consistent it is required to pass the convergence test, is used to assess the consistency and stability of a numerical scheme based on its convergence. As a result, the mesh convergence study became a well-recognized and the standard verification method of CFD codes. The ratio of consecutive error norms is a perfect vehicle to catch any coding error/algorithm problem. The points need to be considered in any mesh-convergence study include:

* For the FDM and FVM descritzations developed under the assumption of smooth function, the discontinuities and jagged initial or boundary conditions can locally or globally decrease the convergence rate. Linfinity[[1]](#footnote-1)should be included as an ultimate diagnostic tool for local errors and worst case scenario. “L2”is more forgiving norm compare to the first error norm “L1”. We recommend L1 as an appropriate global metric of error.[[2]](#footnote-2)
* Convergence ratio in a very coarse grid oscillates around its main value, as the grid size is refined convergence becomes monotone until the mesh size reaches to a point where the machine precision overtakes the truncation error of the numerical scheme, at this point error norms do not change and convergence rate is zero.
* Although the convergence is a reliable warning of a defect, it should not be forgotten that the main goal in practice is a more accurate solver. Therefore the superiority of methods should be assessed both based on convergence and accuracy. Accuracy metrics similarly are error norms as is discussed above, however for evaluating the accuracy error norms should be normalized by an appropriate scale of the solution.
* All of the convergence tests such as MMS, Richardson Extrapolation, could be run by a same driver. The post processing of the convergence test also could carry out with a same code for all the tests.
* Visualization of time evolution of error and results in the solution domain is a decent strategy for debugging in cases the source of inaccuracy is obscure.

Benchmark for calculating error norms

The natural phenomena involving in the real tidal system hydrodynamic and water quality are nonlinear and the analytical solutions of the governing equation are limited to very limited cases.

The general form of ADR equation which is our interest is:



where *A* is Area, *C* is concentration, *u* is velocity, *K* is longitudinal dispersion coefficient, and *R* is the source term (deposition, erosion, lateral inflow and other forms of sources and sinks). The question here is: if one wants to find accuracy and convergence ratio of a scheme in which the analytical solution is unknown (absence of analytical solution is the main motivator towards all numerical methods), what should be done? It is ideal to test a model’s correctness by comparing its numerical results with analytical solutions; however the difficulty is that there is not a general solution for the non-linear IBVP in hydrodynamics. There are some ways to deal with this problem from the simplest to the most sophisticated:

*Comparing with a higher order code/run on dense mesh*: (the benchmark solver must be verified beforehand) the other issue here is the circularity in this method, there must be one verified code available at the beginning if not we hit an impasse.

*Richardson Extrapolation* is the common method for dealing with commercial packages and multidimensional complex systems (Roache and Knupp, 1993), however the drawback is that the method only checks if the solver converges and it is not able to measure where it is converging to.

Difficulties arise in Richardson EXTRAPOLATION???(BC/IC incompatibility?)

*Method of Manufactured Solutions* (MMS) (Wang and Jia, 2009), and *Prescribed Solution Forcing Method* (PSF) (Dee and Da Silva, 1986). The basic concept of the MMS and PSF is to compare the correctness of numerical solvers using an arbitrary manufactured function. MMS and PSF are conceptually following the same idea, although the former is more general than the latter. PSF have been used for the verification cases in which the user can not access the source code to define boundary conditions such as some groundwater codes.

The MMS is a general approach to provide a certain analytical solution of the governing equation for the question of model testing and verification of non-linear numerical solvers in rigorous procedure. Since only the numerical method is to be tested (not the physics of the problem) it would be effective if an arbitrarily made non-linear function can be used in model verification. The exact solution which is manufactured in this method does not need necessarily be realistic (Roache 2009, 2002; Wang et al., 2009) but the authors recommend to chose it the reasonable ranges. We want a benchmark solution that is non-trivial but analytical, and that exercises all ordered derivatives in the error expansion and in all terms.

Let the differential equation be expressed as:

 (2)

in which *L* denotes the differential operators and *u* is the variable to be solved. When a manufactured function *φ* is substituted into the differential equation, one would have:

 (3)

Since *φ* is not the solution of differential equation, the non-zero *R* is obtained analytically. In the solver, the numerical solution of this equation would be forced to converge to *φ* with the analytical forcing term *R* being added to the mathematical equation of the numerical model as the source term. The verification of a numerical model is simple because the solution of equation (1) is known; one needs only compare the difference between the manufactured analytical function, *φ*, and the numerical solution of equation (1). Although the function *φ* can be manufactured arbitrarily, it has to be non-trivial for all the terms of the involved mathematic equations to make a meaningful verification tests. MMS does not require the satisfaction of any particular boundary condition other than those defined by *φ* along the boundaries of computational domain. The difficulties in MMS are the parameters in the equation (1) are need to be checked under the same situation on an estuarine problem because the solver is work in especial ranges of dimensionless numbers also the area and velocity should satisfy the continuity of mass. The following example clarifies the method (Should I provide an example here?):

Scaling of the problem for an estuary

The ADR solver is only working in the feasible ranges of dimensionless numbers () so in case the reaction rate in equation (1) should not exceed a certain limit, and generally speaking the test suit has to be designed within the natural scales of the physical problem. The assumed scales and ranges are as follows: Area~ 1000 [m2], C (0 – 0.05) [vol/vol=1], u (±0.2-2) [m2/s],

1. ,,, where *v*= U num - U exact [↑](#footnote-ref-1)
2. It is proven that kL∞ ≤ L2 ≤ L1 ≤ L∞ where k is a constant and 0<k<1, here norms are assumed to be scaled. [↑](#footnote-ref-2)